

# Problems of Dark Matter

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## Abstract

The data indicating existence of different forms of dark matter in the universe as well as the role of this matter in structure formation are briefly reviewed. It is argued that vacuum energy gives a dominant contribution into the total energy density of the universe. The model of structure formation with unstable tau-neutrino with MeV-mass and KeV-majoron is described.

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It is commonly agreed now that the matter (or energy) in the universe is predominantly (90% or even more) invisible and that most probably this invisible matter is not the normal baryonic stuff that we are familiar with. The agreement however does not go beyond this point and as for the nature of this dark matter the problem remains open and will probably be open for quite some time. There are some hints from cosmology regarding the properties of dark matter. They are given by direct astronomical observations and by the theory of large scale structure formation in the universe. From the other side the elementary particle theory gives plenty predictions about possible new stable particles which might be constituents of dark matter. Unfortunately these predictions do not directly follow from the well established minimal  $SU(3) \times SU(2) \times U(1)$  - theory but from one or other higher energy extensions of this theory which have not yet been verified by experiment. So at the moment we are unable to chose different possibilities and have to rely only on aesthetic properties of the model which may be grossly misleading.

The fraction of a particular form of matter in the universe is given in terms of the dimensionless parameter

$$\Omega_j = \rho_j / \rho_c \quad (1)$$

where  $\rho_j$  is the mass (energy) density of this form of matter and  $\rho_c$  is the critical or closure density,

$$\rho_c = \frac{3H^2 m_{Pl}^2}{8\pi} = 1.88 h_{100}^2 \times 10^{-29} g/cm^3 = 10.5 h_{100}^2 KeV/cm^3 \quad (2)$$

Here  $m_{Pl} = 1.22 \times 10^{19} GeV$  is the Planck mass (the Newtonian gravitational constant is  $G_N = m_{Pl}^{-2}$ ). The Hubble constant  $H$  is parametrized in terms of dimensionless quantity  $h$ :

$$H = 100h km/sec/Mpc \quad (3)$$

It is usually assumed that  $0.4 < h < 1$ , but the recent data [1] favor the values in the upper part of this interval,  $h = 0.75 - 0.85$ . The larger values of  $h$  were rejected by many astronomers because of the universe age problem (see below) but now more and more evidence is accumulated in favor of that and the attitude seems to be gradually changing.

If the Hubble constant and  $\Omega_j$  are known, the universe age can be expressed through them as

$$t_u = \frac{1}{H} \int_0^1 \frac{dx}{\sqrt{1 - \Omega_{tot} + \Omega_m x^{-1} + \Omega_{vac} x^2}} \quad (4)$$

where  $\Omega_m$  and  $\Omega_{vac}$  correspond respectively to the energy density of nonrelativistic matter and to the vacuum energy density (or, what is the same, to the cosmological constant);  $\Omega_{tot} = \Omega_m + \Omega_{vac}$ , and  $1/H = 9.8h_{100}^{-1} \times 10^9 \text{yr}$ .

On the other hand the universe age can be found from nuclear chronology which uses measurements of the ratios of the long-lived isotopes,  $^{187}\text{Re}/^{232}\text{Th}$  or  $^{238}\text{U}/^{235}\text{U}$ , and from the estimated ages of old stellar clusters. Both methods give close results in the range (12 - 20) Gyr. The recent analysis gives  $t_u \approx 18$  Gyr, though somewhat smaller values are possibly not excluded (for the review and the list of references see e.g. paper [2]).

If the universe is dominated by nonrelativistic matter, as usually assumed, the age is approximately equal to  $t_u \approx 9.8h^{-1}/(1 + \sqrt{\Omega_m}/2)$  Gyr. Even with  $\Omega = 0$  the large age,  $t_u = 18 \text{Gyr}$ , and  $h > 0.75$  are inconsistent. The observed tendency to large values of  $H$  and  $t_u$  presents a very strong argument in favor of nonvanishing vacuum energy. For  $\Omega_{tot} = \Omega_m + \Omega_{vac} = 1$  eq.(4) gives

$$t_u = \frac{2}{3H\sqrt{\Omega_{vac}}} \ln \frac{1 + \sqrt{\Omega_{vac}}}{\sqrt{1 - \Omega_{vac}}} \quad (5)$$

With  $h = 0.75$  and  $\Omega_{vac} = 0.8$  we get  $t_u = 14$  Gyr. If this is the case most of invisible energy in the universe is just the energy of empty space, of vacuum. Unfortunately our understanding of vacuum energy is very poor. Any reasonable estimate of it gives the result which is some 50-100 orders of magnitude higher than the observational limit  $\rho_{vac} < 10^{-47} \text{GeV}^4$  (for the review see refs. [3, 4]). For example there are contributions from quark and gluon condensates which are well established in QCD and which are about  $10^{-4} \text{GeV}^4$ . So there must exist a contribution into vacuum energy from something not related to quarks and gluons but with exactly the same magnitude and the opposite sign. It is hard to imagine an accidental cancellation with such an accuracy but no dynamical mechanism has yet been found. To my

mind the best possibility for solving the cosmological constant problem is the adjustment mechanism [5, 6] (for recent papers and some references see [7] and reviews [3, 4]), which ensures a cancellation of vacuum energy by an action of a new field coupled to gravity. This cancellation is generically not complete and a noncompensated amount of  $\rho_{vac}$  is always of the order of  $\rho_c(t)$ . In such models vacuum energy and the energy of the new field are essential at any stage of the universe evolution (in contrast to the models with normal time independent vacuum energy) and this might have an impact on primordial nucleosynthesis, structure formation, etc.

Astronomical data on the amount of different forms of matter in the universe can be summarized as following:

1. The directly seen luminous matter contributes very little to the total mass of the universe:  $\Omega_{lum} = (3.5 \pm 1) \times 10^{-3} h^{-1}$ .
2. Total amount of baryons found from primordial nucleosynthesis is  $\Omega_{bar}^{(NS)} = 4 \times 10^{-3} \eta_{10} h^{-2}$  where  $\eta_{10} = 10^{10}(N_B/N_\gamma)$  with  $\eta_{10} = 1 - 7$ . The recent analysis of the problem can be found e.g. in refs. [8, 9, 10]. At the moment there is no consensus whether  $\eta$  is relatively high, near the upper bound, or near the lower end of the interval. In the first case we should expect plenty of invisible baryons in the universe while in the second all baryons may be visible. To resolve this ambiguity it is very important in particular to find the amount of primordial deuterium. At the moment there are two conflicting sets of observations, one gives by number  $D/H \approx 2 \times 10^{-4}$  [11, 12] and requests a small  $\eta$ ,  $\eta_{10} \approx 1$ , while the other gives  $D/H = (1 - 2) \times 10^{-5}$  [13] and  $\eta_{10} \approx 3$ .
3. Flat rotational curves observed in gravitationally bound systems like gas around galaxies or galactic satellites give, depending on scale,  $\Omega_{rot} = 0.1 - 0.3$ . Rotational velocities are measured for hundreds of galaxies up to distances about 30 Kpc through HI hydrogen gas, up to 100 Kpc for hot X-ray gas around galaxies and up to 200 Kpc for galactic satellites [14, 15, 16]. These flat rotational curves mean that the gravitating

mass is not confined inside the luminous region with the size about 10 Kpc (for large galaxies) but linearly rises with the distance,  $M(r) \sim r$ . No cutoff for this behavior has yet been observed.

4. The recent analysis [17] of peculiar velocities of about 3000 galaxies gives  $\Omega_{lsf} = [b(0.74 \pm 0.13)]^{5/3}$  where  $b$  is the biasing parameter,  $b = (\delta\rho/\rho)_{vis}/(\delta\rho/\rho)_{tot}$ . The value of the latter is not known but we believe that it is close to one and most probably  $b \geq 1$ .
5. The lower limits on the universe age,  $t_U > 12 \text{ Gyr}$ , and on the Hubble constant,  $h > 0.7$ , result in the bound  $\Omega_{matter} < 0.2$ , if cosmological constant is zero.
6. Recent observations [18, 19] of hot X-ray gas in rich galactic clusters showed a surprisingly large fraction of baryons with respect to the total mass of the clusters. If the mass contained in the clusters represents a fair sample for the total mass in the universe then these data together with the nucleosynthesis constraint on  $\Omega_B$  put the strong upper bound on the energy density in the universe:  $\Omega_{clustered} \leq 0.15h_{100}^{-1/2}/(1+0.55h_{100}^{3/2})$ . This bound is evidently valid for the clustered matter and is not applicable for the uniformly distributed one. These data are probably a good indication on nonzero vacuum energy  $\Omega_{vac} \approx 0.8$ . It is uniformly distributed and for the typical cluster size about 1 Mpc does not contribute much inside this radius.
7. Inflationary universe model predicts  $\Omega = 1 \pm 10^{-4}$ . Recently there appeared attempts [20] to reconcile inflation with  $\Omega \neq 1$ , which were stimulated by the new astronomical data. These attempts however do not look as natural as the original inflationary prediction since they request a tuning of inflationary and postinflationary stages in such a way that  $\exp(2H_I\tau) = (T_R/T_0)^2(\Omega_0^{-1} - 1)^{-1}$ , where  $T_R$  is the reheating temperature after inflation,  $T_0 = 2.7 \text{ K}$  is the present-day temperature of the cosmic microwave radiation, and  $\tau$  is the duration of inflation. This condition looks very

strange because physically these epochs are not related. To my mind  $\Omega = 1$  remains a very strong prediction of inflationary cosmology.

At the present stage one cannot definitely claim what is the correct value of  $\Omega$ . I think that a nonzero value of  $\rho_{vac}$  should be seriously considered. To my mind the best choice is the following:  $\Omega_{bar} \approx \Omega_{vis} \approx (0.3 - 0.5)\%$ ;  $\Omega_{DM} = 0.1 - 0.2$ , and  $\Omega_{vac} = 0.8 - 0.9$  with  $\Omega_{tot} = 1$ .

Even if one believes that vacuum energy contributes (80-90)% to the total energy density in the universe, still there should be some other unknown form of invisible matter which provides flat rotational curves, large scale flows and contributes the remaining (10-20)% into  $\Omega_{tot}$ . Nucleosynthesis constraint does not permit this matter to be baryonic and we do not know what it is. The only hints about its nature come from elementary particle theory and from the theory of large scale structure formation. Minimal supersymmetric extensions of the standard model predict existence of a massive stable particle with  $m = O(100 GeV)$ . The calculated cosmic abundance of these particles naturally give  $\Omega_{LSP}$  close to unity (here LSP stands for lightest supersymmetric particle). This coincidence is quite impressive because apriori one could expect  $\Omega_{LSP}$  different from 1 by many orders of magnitude. This makes LSP a very good candidate for dark matter particle (for a recent review see [21]).

Still one may express some doubts regarding this possibility. Stability of LSP is ensured by R-parity which is conserved in simple supersymmetric models. However we know from the history of particle physics during last half of century that the only conservation laws survived which were protected by a well justified theoretical principle, like e.g. CPT-theorem or electric charge conservation (protected by gauge invariance in QCD). As for R-parity no such principle naturally appeared and thus one should expect that it is nonconserved. In that case LSP's are unstable and if their life-time is cosmologically small, we have to look for other possible candidates for dark matter particles. These could be axions, massive neutrinos, or maybe some other more exotic forms of matter. A classical field which probably compensates nonzero vacuum energy is a very interesting possibility. Models with broken R-

parity which may kill stable LSP's, simultaneously give birth to another possible candidate for dark matter. R-parity breaking is realized through spontaneous breaking of leptonic charge conservation [22, 23] associated with global symmetry group  $U(1)_L$ . Spontaneous breaking of a global symmetry group gives rise to appearance of a (pseudo)goldstone boson, majoron. If there is also an explicit symmetry breaking, the majorons would become massive and, if cosmologically long-lived, could be constituents of dark matter [24, 28].

Some of dark matter candidates, like e.g. massive stable neutrinos, may be rejected on the basis of the theory of large scale structure formation. However not all the basic assumptions of this theory are solid enough to make really strong conclusions. While practically everybody agrees that the universe structure has been formed as a result of evolution of initially small fluctuations under the action of gravitational forces, there is no consensus about the shape of the spectrum of initial density perturbations. Introducing dimensionless quantity  $\delta = \delta\rho/\rho$  we can present the power spectrum in the form

$$\langle \delta^2(x) \rangle = \int dk f^2(k) \quad (6)$$

(assuming that Fourier amplitudes are delta-correlated; one more assumption which may be questioned). Usually for  $f(k)$ , the most simple form is taken namely it is assumed that it does not contain any dimensional parameter,  $f(k)^2 \sim 1/k$ . This corresponds to flat or scale-free spectrum proposed by Harrison [29] and Zeldovich [30]. Inflationary models gave theoretical justification to this form of spectrum and now it is commonly used as the basic spectrum in calculations of structure formation. Sometimes as a simple generalization an arbitrary power law spectrum is considered,  $f^2 \sim 1/k^n$ . The spectrum with  $n \neq 1$  may appear in some inflationary models [31] but with  $n$  not much different from unity. If we permit in principle an existence of primordial spectrum with an arbitrary  $n$ , introducing in this way a new scale to the theory, a combination of several terms with different  $n$  as well as a more complicated functions are also permitted but without a guiding principle for choosing a particular form of the spectrum, the theory would completely loose its predictive power.

So we have to keep in mind that the judgement on the possible dark matter particles is made under assumption that the spectrum of perturbations has simple scale-free form.

The simplest model which was successfully used for description of structure formation until recently was the one with a single component cold dark matter particles (like LSP or axion) and flat spectrum of initial perturbations (for the review see e.g. refs. [32, 33]). The model reasonably well described galaxy distribution at the scale of tens megaparsec with the only free parameter, the overall normalization of the spectrum. However the COBE measurements [34] has fixed the normalization at large scale end of the spectrum and with this normalization the predictions at smaller scales became approximately twice above the observations. An evident possible cure is to change the shape of the spectrum of primordial fluctuations. It seems to be a very interesting possibility but since this idea is not supported by the inflationary scenario, the major line of investigation goes along consideration of different forms of dark matter with the same flat spectrum of density perturbations or maybe an introduction of the cosmological constant. The basic idea of all these models was to suppress the power of the evolved spectrum at smaller scales relative to that at larger (COBE) scales.

One possibility is to shift the epoch of matter dominance (MD) to a later stage in a simple cold dark matter model. Since the characteristic scale at which perturbations started to rise in this model is determined by the horizon size at the onset of the MD stage, shifting it to a later moment gives less time for rising of the fluctuations and correspondingly less power at galactic and cluster scales. This goal can be achieved if one assumed that universe is open so that  $h_{100}^2 \Omega \approx 0.2$ . However the low value of  $\Omega$  is disfavoured by inflationary scenarios which (at least in simple versions) predict  $\Omega = 1$ . One can recover this prediction of inflation in the universe with low matter density if the cosmological constant  $\Lambda$  (or in other words, vacuum energy) is nonzero. As we have mentioned above the recent data indicating a rather high value of the Hubble constant,  $h_{100} \approx 0.8$ , support the idea of nonzero  $\Lambda$  with the fraction of the vacuum energy  $\Omega_{vac} = 0.8$ . The models with nonzero  $\Lambda$  give a satisfactory description of the observed structure [35] with flat spectrum and COBE normalization.



A mixed (hot+cold) dark matter scenario can also do the necessary job of diminishing the power at small scales because (initially flat) perturbations in hot dark matter are erased at scales smaller than  $10^{14}M_{\odot}$  by free streaming if the dark matter is collisionless as is the case of neutrinos. A good description of the structure requests 70% of CDM and 30% of HDM [32]. It gives even better description if there are two equal mass neutrino species each with the mass 2.5 eV [36] as is suggested by the recent indications of neutrino oscillations by the Los Alamos group.

Recently there appeared a renewed interest to the idea of structure formation with unstable particles [37]. It is assumed in these models that there exists a massive long-lived particle, usually tau-neutrino with the mass in MeV range which decayed into massless species at the epoch when the mass density of the parent particles dominated the energy density of the universe. Correspondingly the present-day energy density of relativistic particles would be bigger than in the standard scenario and the onset of MD stage would take place later.

The common shortcoming of these models is that they all demand a certain amount of fine-tuning. Generally one would not expect that the contribution from hot and cold dark matter into the universe mass are about the same, they may differ by many orders of magnitude. One would also suspect that the vacuum energy which remains constant in the course of the universe expansion is by no means related to the critical energy now, which goes down with time as  $m_{Pl}^2/t^2$ . However adjustment mechanism [5, 6] may possibly solve this problem. The models with unstable particles mentioned above are also based on the assumption of two independent components with a close contribution into  $\Omega$ : the massive unstable particles themselves and unrelated cold dark matter. This is definitely unnatural and this shortcoming stimulated search for other models. Recently in ref. [38] a return to the universe with a single cold dark matter component (of course except for baryons) was advocated. For successful description of the observed structure the authors need the power index of the spectrum  $n = 0.8 - 0.9$ , a low value of the Hubble constant,  $h = 0.45 - 0.5$ , and a large contribution of tensor perturbations (gravitational waves) into quadrupole fluctuation

of background radiation temperature,  $C_T/C_S = 0.7$ .

The old idea of mirror world [25] which is coupled to ours (almost) only through gravity was recently revived [26, 27] for possible explanation of the puzzles in neutrino physics and as a possible source of dark matter in the universe. The model of ref.[26] predicts an existence of a heavy sterile neutrino with mass in KeV range which might be a warm dark matter particle and a light (but massive) usual neutrino with mass about 10 eV for a hot dark matter particle. The relic abundances of these two neutrinos are predicted to be close to each other.

Another attempt to overcome unnaturalness of multicomponent dark matter model has been done in paper [28]. In this paper a model is considered in which unstable particles and the particles of cold (or possibly warm) dark matter are closely connected. In fact the decay of the former produces particles of the present-day dark matter. A necessary background model of this kind in particle physics was proposed some time ago [22, 23] as an attempt to find a phenomenologically acceptable description of R-parity breaking. The underlying mechanism is the spontaneous breaking of leptonic charge conservation at electroweak scale. The model contains a Majorana type tau-neutrino with the mass around MeV which decays into massive but light majoron  $J$  with mass in KeV region. Life-time with respect to this decay, as estimated in refs. [22], could be of order of years depending on the values of parameters, i.e. in the interval interesting for structure formation. It is worth noting that there is no stable SUSY particle in this model so the dark matter cannot be associated with it but the model itself produce a candidate for dark matter, namely a massive majoron. The model possesses some features like sufficiently large diagonal coupling of  $\nu_\tau$  to majorons or selfinteraction of majorons which rather naturally permit to resolve appearing cosmological problems in particular the problem of extra massless particle species during primordial nucleosynthesis. The cosmological properties of this model are rather unusual and are interesting by themselves. Dark matter particles (majorons) in this model are strongly self-interacting and thus the structure formation in this model is different from

the traditional one with collisionless dark matter particles [39, 40, 41, 28]. In particular the shape of galactic halo would depend on the dark matter selfinteraction. The recent data [42] might give an indication that collisionless dark matter gives a poor description of the shape of halo in dwarf galaxies.

At first sight the model of ref. [28] encounters very serious cosmological difficulties. Tau-neutrino with the mass in MeV range and stable at nucleosynthesis time scale would strongly distort primordial abundance of light elements [43, 44] and should be excluded. However the Yukawa coupling of  $\nu_\tau$  to majorons helps to reduce the number density of tau-neutrinos down to a safe value through the reaction  $2\nu_\tau \rightarrow 2J$ . In this model the effective number of relativistic degrees of freedom during nucleosynthesis may be even below 3 in agreement with the recent claims [8, 45].

However a strong reduction of the number density of  $\nu_\tau$  at nucleosynthesis through the annihilation into majorons produces the equilibrium amount of majorons such that the ratio of their number density to that of photons becomes  $n_J/n_\gamma = 0.5(T_J/T_\gamma)^3$ . By Gerstein-Zeldovich bound [46, 47] the mass of such particles cannot exceed roughly speaking 30 eV. On the other hand for succesful structure formation we need  $m_J = O(KeV)$  and it looks as if the model does not work. Fortunately this is not so because of relatively strong majoron selfcoupling:  $\lambda J^4$  with  $\lambda = 0.1 - 0.01$  [24]. This interaction gives rise to majoron cannibalism through the process:  $4J \rightarrow 2J$ . This process is fast enough to reduce the majoron number density by an order of magnitude [28] and to permit the majorons to have the mass appropriate for warm dark matter particles.

To conclude we definitely know that the world predominantly consists of matter which is different from what we directly see around. We do not know what kind of matter it is. Most probably this unknown matter consists of several different components. Why these components have similar energy densities in the present-day universe is also unknown. One of the component is very probably just energy of vacuum. The problem of vacuum energy is, to my mind, the central one in cosmology and particle physics in particular in connection

with dark matter. A new massless field coupled only to gravity may solve the cosmological constant problem and to contribute to the dark matter. As for more traditional components which should be also present, even if vacuum energy is nonvanishing, the search for low energy supersymmetry is very important. In particular it is essential to know if R-parity is indeed conserved and the lightest supersymmetric particle is stable. An improvement of accuracy in determination of tau-neutrino mass may have an important impact on cosmology. A related phenomenon is the primordial nucleosynthesis because the produced abundance of light elements depend on  $\nu_\tau$  mass if it is in MeV range. Hopefully in the nearest future the ambiguity in determination of the amount of primordial deuterium, which is very sensitive to the total density of baryons will be settled down. If one is optimistic one may hope that the mystery of building blocks of the universe will be resolved before the end of the next (or this, if one is superoptimistic) century.

## Acknowledgments

This work was supported by DGICYT under grants PB92-0084 and SAB94-0089 (A. D.).

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